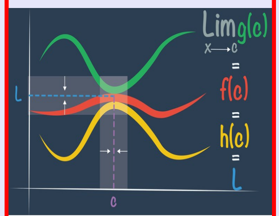
Calculus I

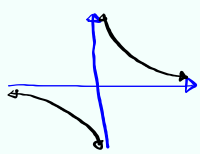
Lecture 19



Feb 19-8:47 AM

Limits at $\pm\infty$

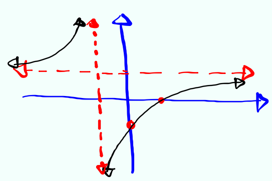
$f(x) = \frac{1}{x}$



$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

$f(x) = \frac{x-1}{x+1}$



$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to \infty} \frac{x-1}{x+1} = \frac{\infty}{\infty}$ I.F.

Divide everything by the highest power of x.

$$\lim_{x \to \infty} \frac{\frac{x-1}{x}}{\frac{x+1}{x}} = \lim_{x \to \infty} \frac{\frac{x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x \to \infty} \frac{1 - \frac{1}{x} \to 0}{1 + \frac{1}{x} \to 0}$$

$$= \frac{1-0}{1+0} = \boxed{1}$$

Oct 1-7:28 AM

$$\lim_{x\to\infty} \frac{2x^2-3x+4}{3x^2+2x-8} = \frac{\infty}{\infty} \text{ I.F.}$$

Divide everything by x^2

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}-\frac{3x}{x^2}+\frac{4}{x^2}}{\frac{3x^2}{x^2}+\frac{2x}{x^2}-\frac{8}{x^2}} = \lim_{x\to\infty} \frac{2-\cancel{\frac{3}{x}}^{\to 0}+\cancel{\frac{4}{x^2}}^{\to 0}}{3+\cancel{\frac{2}{x}}^{\to 0}-\cancel{\frac{8}{x^2}}^{\to 0}} = \boxed{\frac{2}{3}}$$

Oct 1-7:37 AM

$$\lim_{x\to\infty} x \sin\cancel{\frac{1}{x}}^{\to 0} \to 0 = \infty \cdot 0 \quad \text{I.F.}$$

I.F. $\to \frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty \cdot 0$

$$\lim_{x\to\infty} x \sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

Let $h = \frac{1}{x}$

as $x \to \infty$, $h \to 0$

$$= \lim_{h\to 0} \frac{\sin h}{h} = \boxed{1}$$

Oct 1-7:41 AM

$$\lim_{x\to\infty} \frac{2x+3}{\sqrt{x^2-4}} = \frac{\infty}{\infty} \quad \text{I.F.}$$

$x = \sqrt{x^2}$ is $x \geq 0$ highest power for x is 1

Divide everything by x

$$\lim_{x\to\infty} \frac{\frac{2x+3}{x}}{\frac{\sqrt{x^2-4}}{x}} = \lim_{x\to\infty} \frac{\frac{2x+3}{x}}{\frac{\sqrt{x^2-4}}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{2+\cancel{\frac{3}{x}}\to 0}{\sqrt{1-\cancel{\frac{4}{x^2}}\to 0}}$$

$$= \frac{2}{\sqrt{1}} = \boxed{2}$$

Oct 1-7:45 AM

Evaluate $\displaystyle\lim_{x\to\infty} \frac{4x-x^2}{\sqrt{4x^2+6}} = \frac{-\infty}{\infty}$ I.F.

$x^2 = \sqrt{x^4}$

as $x \to \infty$

Divide by x^2

$$\lim_{x\to\infty} \frac{\frac{4x}{x^2} - \frac{x^2}{x^2}}{\sqrt{\frac{4x^2+6}{x^4}}} = \lim_{x\to\infty} \frac{\cancel{\frac{4}{x}}\to 0 - 1}{\sqrt{\cancel{\frac{4}{x^2}}\to 0 + \cancel{\frac{6}{x^4}}\to 0}}$$

$= \frac{-1}{0}$ undefined

Use a graphing Software and graph $f(x) = \dfrac{4x-x^2}{\sqrt{4x^2+6}}$ and explore as $x \to \infty$

Oct 1-7:50 AM

For function $f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ is the limit exists.

F-Prime of x

First derivative of $f(x)$

Slope of the tan. line at any point on the graph of $f(x)$

Oct 1-7:57 AM

$f(x) = x^2 - 4x$

Find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) - x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-4x} - 4h \cancel{-x^2} \cancel{+4x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 4)}{\cancel{h}} = \lim_{h \to 0} (2x + h - 4) = \boxed{2x - 4}$$

Oct 1-7:59 AM

$f(x) = \sqrt{x}$

find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{\cancel{x} + \overset{1}{\cancel{h}} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}} = \frac{\sqrt{x}}{2x}$$

Oct 1-8:03 AM

find $f'(x)$ for $f(x) = \frac{1}{x}$, then evaluate $f'(\frac{1}{2})$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

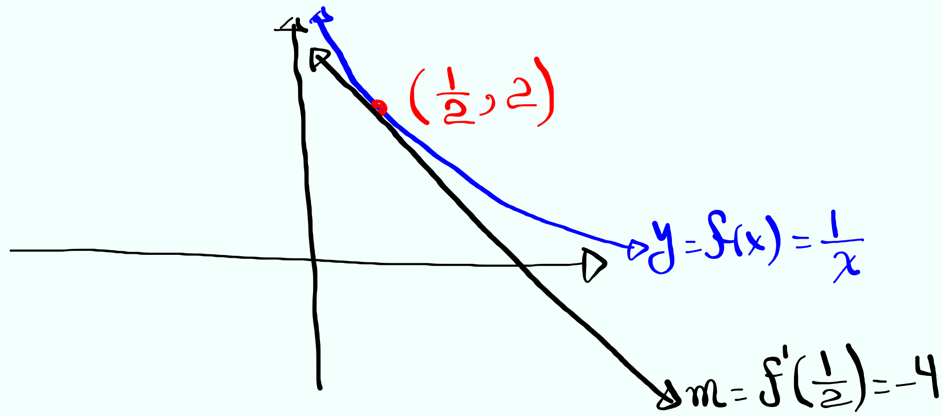
$LCD = x(x+h)$

$$= \lim_{h\to 0} \frac{x\cancel{(x+h)} \cdot \frac{1}{\cancel{x+h}} - \cancel{x}(x+h) \cdot \frac{1}{\cancel{x}}}{h\, x(x+h)}$$

$$= \lim_{h\to 0} \frac{\cancel{x} - \cancel{x} - \overset{-1}{\cancel{h}}}{\cancel{h}\, x(x+h)} = \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{\frac{-1}{x^2}}$$

$$f'(\tfrac{1}{2}) = \frac{-1}{(\frac{1}{2})^2} = \frac{-1}{\frac{1}{4}} = \boxed{-4}$$

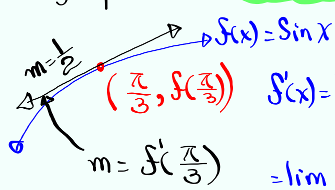
Oct 1-8:07 AM



First derivative $f'(x)$ is the slope of the tan. line at $(x, f(x))$

Oct 1-8:14 AM

Find the slope of the tan. line to the graph of $f(x) = \sin x$ at $x = \frac{\pi}{3}$.



$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \sin x \cdot \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \cdot \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1} = \boxed{\cos x}$$

$f(x) = \sin x$, $f'(x) = \cos x$ $\quad m = f'(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

Oct 1-8:16 AM

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x) = x^2 - 4x$

$$f'(a) = \lim_{x \to a} \frac{x^2 - 4x - a^2 + 4a}{x - a}$$

$\frac{0}{0}$ I.F.

$$= \lim_{x \to a} \frac{x^2 - a^2 - 4x + 4a}{x - a} = \lim_{x \to a} \frac{(x+a)(x-a) - 4(x-a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x + a - 4)}{\cancel{x - a}}$$

$$= \lim_{x \to a} (x + a - 4) = a + a - 4$$

$$= \boxed{2a - 4}$$

Oct 1-8:25 AM

Find $f'(x)$ for $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{(\cancel{x} + h - \cancel{x})((x+h)^2 + (x+h)x + x^2)}{h}$$

$$= \lim_{h \to 0} \left[(x+h)^2 + (x+h)x + x^2 \right] = x^2 + x \cdot x + x^2$$

$$= \boxed{3x^2}$$

Oct 1-8:30 AM